$$\int_0^{\pi/2} \sin^n x \, dx$$

let $I_n = \int_0^{\pi/2} \sin^n x\, dx$

we have $I_n = -\dfrac{\sin^{n-1} x \cos x}{n} + \dfrac{(n-1)}{n} I_{n-2}$

$\because \left[\cos\left[\frac{\pi}{2}\right] = 0 \ \& \ \sin(0) = 0\right]$

$\therefore I_n = \dfrac{(n-1)}{n} I_{n-2}$

from above eqn, we have

$I_{n-2} = \dfrac{n-3}{n-2} I_{n-4}$

$I_{n-4} = \dfrac{n-5}{n-4} I_{n-6}$

$I_{n-6} = \dfrac{n-7}{n-6} I_{n-8}$

. . . . . .

$I_3 = \dfrac{2}{3} I_1$

$I_2 = \dfrac{1}{2} I_0$

$\because I_n = \dfrac{(n-1)}{n} I_{n-2}$

$= \dfrac{(n-1)}{n} \dfrac{(n-3)}{(n-2)} I_{n-4}$

$= \dfrac{(n-1)}{n} \dfrac{(n-3)}{(n-2)} \dfrac{(n-5)}{(n-4)} I_{n-6}$

. . . . . .

$$I_n = \begin{cases} \dfrac{(n-1)}{n} \dfrac{(n-3)}{(n-2)} \dfrac{(n-5)}{(n-4)} \cdots \dfrac{2}{3} I_1 & \text{if } n \text{ is odd} \\[2ex] \dfrac{(n-1)}{n} \dfrac{(n-3)}{(n-2)} \dfrac{(n-5)}{(n-4)} \cdots \dfrac{1}{2} I_0 & \text{if } n \text{ is even} \end{cases}$$

Here $I_1 = \int_0^{\pi/2} \sin^1 x\, dx = -\cos x\, dx\Big]_0^{\pi/2}$

$= -[0 - 1] = 1$

$I_0 = \int_0^{\pi/2} \sin^0 x\, dx = \int_0^{\pi/2} dx$

$= x\Big]_0^{\pi/2}$

$= \pi/2$

$$\therefore I_n = \begin{cases} \dfrac{(n-1)}{n} \dfrac{(n-3)}{(n-2)} \dfrac{(n-5)}{(n-4)} \cdots \dfrac{2}{3} \cdot 1 & n \text{ is odd} \\[2ex] \dfrac{(n-1)}{n} \dfrac{(n-3)}{(n-2)} \dfrac{(n-5)}{(n-4)} \cdots \dfrac{1}{2} \cdot \dfrac{\pi}{2} & \text{if } n \text{ is even} \end{cases}$$